# Rossmoyne Senior High School

**Semester Two Examination, 2020**

**Question/Answer booklet**

## MATHEMATICS APPLICATIONS UNITS 3&4
**Section Two:**
**Calculator-assumed**

**SOLUTIONS**

WA student number:    In figures [ ][ ] [ ][ ][ ] [ ][ ][ ]

In words _____

## Time allowed for this section
Reading time before commencing work:    ten minutes
Working time:    one hundred minutes

Number of additional answer booklets used (if applicable): [ ]

## Materials required/recommended for this section
***To be provided by the supervisor***
This Question/Answer booklet
Formula sheet (retained from Section One)

***To be provided by the candidate***
Standard items:    pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items:    drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

## Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---|---|---|---|---|---|
| Section One: Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
| | | | | **Total** | 100 |

## Instructions to candidates

1.   The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2.   Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3.   You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4.   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5.   It is recommended that you do not use pencil, except in diagrams.

6.   Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7.   The Formula sheet is not to be handed in with your Question/Answer booklet.
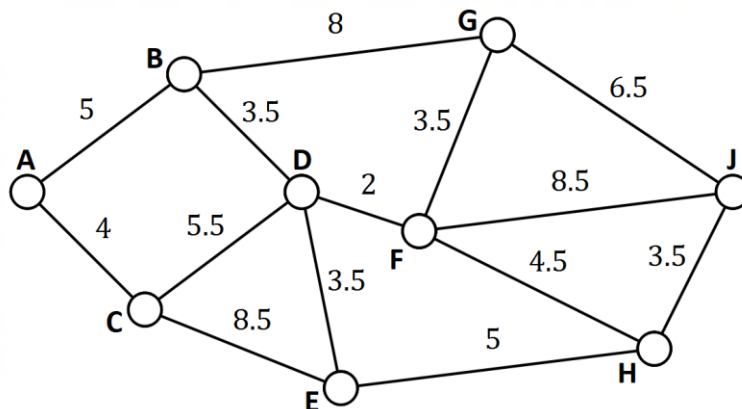
**Section Two: Calculator-assumed** **65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

**Question 9** **(5 marks)**

The weights on the edges of the graph below are the flight times in hours between adjacent airports, represented by the vertices labelled A - J.



(a) Determine the minimum total flight time between airport A and airport J. (2 marks)

| Solution |
| --- |
| Airports $A, B, G, J$: 19.5 hours. |
| Airports $A, B, D, F, H, J$: 18.5 hours. |
| Airports $A, C, E, H, J$: 21 hours. |
| |
| Minimum time is 18.5 hours. |
| **Specific behaviours** |
| ✓ indicates correct route or 2 incorrect |
| ✓ minimum time in hours |

(b) When planning a journey, a traveller allows 45 minutes at each airport on their route, including the **first and last**, to allow for check in, transfers, baggage collection and so on. Determine the quickest route for this traveller from airport A and airport J and state their total journey time. (3 marks)
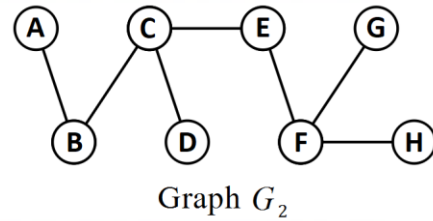
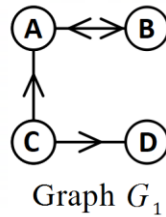| Solution |
| --- |
| Airports $A, B, G, J$: $19.5 + 4 \times 0.75 = 22.5$ hours. |
| |
| Airports $A, B, D, F, H, J$: $18.5 + 6 \times 0.75 = 23$ hours. |
| |
| Quickest route: Airports $A, B, G, J$. |
| Total journey time is 22.5 hours. |
| |
| **Specific behaviours** |
| ✓ indicates one route |
| ✓ indicates one journey time |
| ✓ correct journey time for ABGJ |

**Question 10** **(6 marks)**

(a) Connected graphs $G_1$ and $G_2$ are shown at right.

The adjacency matrices for $G_1$ and $G_2$ are $M_1$ and $M_2$ respectively.

Graph $G_1$

Graph $G_2$

(i) Construct matrix $M_1$. (2 marks)

| Solution |
|---|
| $M_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ |
| **Specific behaviours** |
| ✓ square $4 \times 4$ matrix |
| ✓ correct matrix |

(ii) Determine the number of entries in $M_2$ that are 0. (2 marks)

| Solution |
|---|
| $M_2$ will have $8 \times 8 = 64$ entries. There are 7 edges in $G_2$ and each will lead to two 1's in $M_2$ and so that will leave $64 - 2 \times 7 = 50$ entries that are 0 in $M_2$. |
| **Specific behaviours** |
| ✓ indicates $M_2$ will have $64$ entries |
| ✓ correct number |

(b) The adjacency matrix for graph $G_3$ is $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 2 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix}$.

State, with justification, whether $G_3$ is a simple graph. (2 marks)

| Solution |
|---|
| No - there are multiple edges between some vertices (between $v_2, v_3$ and $v_4, v_2$). |
| **Specific behaviours** |
| ✓ states no with some justification |
| ✓ correct justification |

**Question 11** **(6 marks)**

Using the declining balance method of depreciation, the value of an industrial machine at the end of its first year of use (year 1) is \$63 000 and at the end of year 2 is \$55 125.

The value of the machine in dollars at the end of year $n$ can be modelled by the recurrence relation $V_{n+1} = 0.875V_n$, $V_1 = 63\ 000$.

(a) Show mathematically how to derive the value 0.875 in the recurrence relation from information in the question. (1 mark)

| Solution |
|---|
| $55\ 125 \div 63\ 000 = 0.875$ |
| **Specific behaviours** |
| ✓ shows quotient |

(b) Write the rule for the $n^{\text{th}}$ term of this sequence. (1 mark)

| Solution |
|---|
| $V_n = 63\ 000(0.875)^{n-1}$ |
| **Specific behaviours** |
| ✓ correct formula |

(c) At the end of which year is the machine first valued at less than \$10 000? Justify your answer. (2 marks)

| Solution |
|---|
| At the end of year 15. |
| $V_{14} = 11\ 103.12$ but $V_{15} = 9\ 715.23$. |
| **Specific behaviours** |
| ✓ correct year |
| ✓ shows terms either side of \$10 000 or $V_{15}$ |

(d) Determine the value of the machine when it was new (at the start of the first year) and hence calculate its total decline in value, to the nearest dollar, over its first eight years of use. (2 marks)

| Solution |
|---|
| $V_0 = 63\ 000 \div 0.875 = \$72\ 000$ |
| $V_0 - V_8 = 72\ 000 - 24\ 739.84$ |
| $= \$47\ 260.16$ |
| Total decline in value is \$47 260. |
| **Specific behaviours** |
| ✓ initial value |
| ✓ total decline |

**See next page**

**Question 12** (10 marks)

A factory operates three consecutive eight-hour shifts A, B and C each day. The table below shows the number of workers who turned up late for each shift, together with a three-point moving average $m$.

| Time period $t$ | Day | Shift | Number late | Moving average $m$ |
|---|---|---|---|---|
| 1 | Mon | A | 48 | – |
| 2 | Mon | B | 60 | $P$ |
| 3 | Mon | C | 54 | 55 |
| 4 | Tue | A | 51 | 58 |
| 5 | Tue | B | 69 | 59 |
| 6 | Tue | C | 57 | 60 |
| 7 | Wed | A | 54 | 60 |
| 8 | Wed | B | $Q$ | 61 |
| 9 | Wed | C | 60 | – |

(a) Briefly describe the purpose of calculating a set of moving averages for a time series.
(1 mark)

| Solution |
|---|
| To smooth time series data and expose the underlying trend. |

| Specific behaviours |
|---|
| ✓ indicates smoothing |

(b) Determine the value of $P$ and the value of $Q$ in the table above. (2 marks)

| Solution |
|---|
| $P = (48 + 60 + 54) \div 3 = 54$ |
| $61 = (54 + Q + 60) \div 3 \Rightarrow Q = 69$ |

| Specific behaviours |
|---|
| ✓ value of $P$ |
| ✓ value of $Q$ |

(c) Determine the centred six-point moving average for $t = 4$. (2 marks)

| Solution |
|---|
| $\left(\dfrac{48}{2} + 60 + 54 + 51 + 69 + 57 + \dfrac{54}{2}\right) \div 6 = 57$ |

| Specific behaviours |
|---|
| ✓ indicates appropriate method to centre |
| ✓ correct average |

**See next page**

(d)    Determine the least-squares line to predict $m$ from $t$ using the three-point moving average given in the table. (2 marks)

| Solution |
|---|
| $m = 1.18t + 52.25$ |
| **Specific behaviours** |
| ✓ slope, to at least 2 dp |
| ✓ intercept, to at least 1 dp |
| *NB Using CAS, first entry for $t$ must be 2.* |

Two of the seasonal indices for the above time series are shown in the table below.

| Shift | A | B | C |
|---|---|---|---|
| Seasonal index | 0.88 | | 0.98 |

(e)    Calculate the seasonal index for shift B. (1 mark)

| Solution |
|---|
| $SI_B = 3 - 0.88 - 0.98 = 1.14$ |
| **Specific behaviours** |
| ✓ value of index |

(f)    Forecast the number of late workers for the next A shift (on Thursday), using the least-squares line from (d) and making any necessary seasonal adjustment. (2 marks)

| Solution |
|---|
| $t = 10$ |
| $m = 1.18(10) + 52.25$ |
| $= 64.05$ |
| Forecast $64.05 \times 0.88 = 56$ late workers. |
| **Specific behaviours** |
| ✓ value using least-squares line |
| ✓ correct forecast, as whole number |

**Question 13**        **(7 marks)**

A person has a credit card account with an outstanding debt of \$3 244 and the card provider charges interest at a rate of 16.29% per annum compounded daily.

(a) Determine their card debt in 28 days' time if the card is not used for any more purchases and no repayments are made.        (2 marks)

| N | 28 |
|---|---|
| I% | 16.29 |
| PV | 3244 |
| PMT | 0 |
| FV | −3284.78364 |
| P/Y | 365 |
| C/Y | 365 |

| Solution |
|---|
| $$F = 3\,244\left(1 + \frac{16.29}{100 \times 365}\right)^{28}$$ $$= \$3\,284.78$$ |
| **Specific behaviours** |
| ✓ indicates method (possibly a financial calculator) |
| ✓ correct debt |

The person can pay off their card debt using an unsecured loan from their bank at an interest rate of 16.5% compounded quarterly.

(b) Use effective interest rates to determine, with reasoning, whether the unsecured loan would be a better option for this person.        (3 marks)

| Solution |
|---|
| Card: $EIR = 17.69\%$ p.a. |
| Loan: $EIR = 17.55\%$ pa.. |
| The loan is a better option as the effective interest rate is lower. |
| **Specific behaviours** |
| ✓ one correct rate (accept decimals) |
| ✓ both correct rates (accept decimals) |
| ✓ explains why loan is a better option |

The person chose to pay off their card debt in full by taking out an 18 month secured loan for \$3 244 from a lender who compounds interest monthly. At the end of this time, the person must repay the principal and interest, a sum of \$3 629.

(c) Determine the interest rate charged by this lender.        (2 marks)

| N | 18 |
|---|---|
| I% | 7.500006471 |
| PV | 3244 |
| PMT | 0 |
| FV | −3629 |
| P/Y | 12 |
| C/Y | 12 |

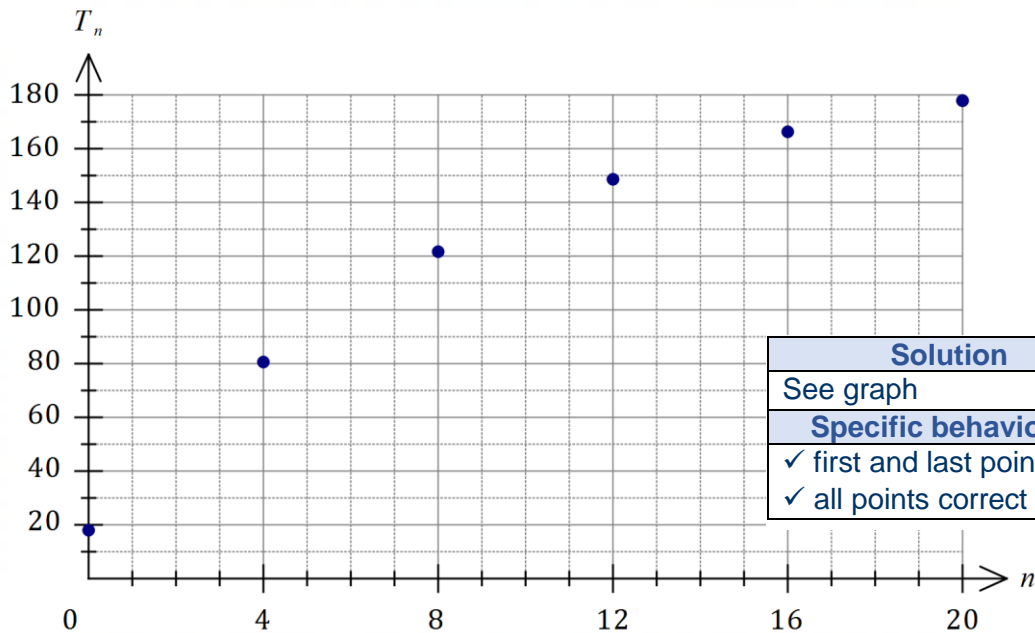| Solution |
|---|
| $$3\,629 = 3\,244\left(1 + \frac{r}{100 \times 12}\right)^{18}$$ $$r = 7.5\% \text{ or } 0.075$$ |
| **Specific behaviours** |
| ✓ indicates method |
| ✓ correct rate (accept decimal) |

**Question 14** **(7 marks)**

The temperature $T_n$, in degrees Celsius, of an oven $n$ minutes after being turned on can be modelled by the recurrence relation $T_{n+1} = 0.9T_n + 20$, $T_0 = 18$.

(a) Use the recurrence relation to complete the following table to the nearest degree Celsius.
(2 marks)

| $n$ | 0 | 4 | 8 | 12 | 16 | 20 |
|-----|---|---|---|----|----|----|
| $T_n$ | **18** | **81** | **122** | **149** | **166** | **178** |

| Solution |
|----------|
| See table |

| Specific behaviours |
|---------------------|
| ✓ at least three correct values; ✓ all correct |

(b) Plot the temperature of the oven at four-minute intervals on the axes below. (2 marks)



| Solution |
|----------|
| See graph |

| Specific behaviours |
|---------------------|
| ✓ first and last points |
| ✓ all points correct |

(c) State the value of $n$ for which the temperature of the oven first exceeds $190\,°C$. (1 mark)

| Solution |
|----------|
| $n = 28$ |

| Specific behaviours |
|---------------------|
| ✓ correct value |

(d) Explain how the temperature of the oven changes in the long term. (2 marks)

| Solution |
|----------|
| The temperature increases become smaller and smaller as the temperature tends towards $200\,°C$. |

| Specific behaviours |
|---------------------|
| ✓ indicates tends to steady state |
| ✓ steady state temperature |

**See next page**

**Question 15**                                         **(7 marks)**

On 1 May 2020 Jem started a new job with an annual salary of $84 000. At that time, the balance of her superannuation fund from previous jobs was $65 240. Jem's new employer deposits a sum equal to 9.5% of her monthly salary into her fund on the last day of each month.

Interest on the balance of an individual's superannuation fund is added on the last day of each month, just before any deposits are made, and the fund advertises an interest rate of 6.6% per annum.

(a)      Determine the balance of Jem's superannuation fund on 1 June 2020.      (3 marks)

| N | 1 |
| I% | 6.6 |
| PV | 65240 |
| PMT | −665 |
| FV | −64933.82 |
| P/Y | 12 |
| C/Y | 12 |

| Solution |
|---|
| Interest multiplier: $1 + 0.066 \div 12 = 1.0055$. |
| Deposit: $84\,000 \div 12 \times 0.095 = \$665$. |
| New balance: |
| $$65\,240 \times 1.0055 + 665 = 65\,240 + 358.82 + 665$$ |
| $$= \$66\,263.82$$ |
| **Specific behaviours** |
| ✓ interest multiplier (or interest amount) |
| ✓ deposit |
| ✓ correct balance |

(b)      Write a recursive relation for the balance $T_n$ of Jem's superannuation fund $n$ months after she started her new job.      (2 marks)

| Solution |
|---|
| $$T_{n+1} = T_n \times 1.0055 + 665, \qquad T_0 = 65\,240$$ |
| **Specific behaviours** |
| ✓ recursive part |
| ✓ initial term $T_0$ |

(c)      Calculate the expected increase in the balance of Jem's superannuation fund after she has been in her new job for two years if her circumstances do not change.      (2 marks)

| N | 24 |
| I% | 6.6 |
| PV | 65240 |
| PMT | 665 |
| FV | −91430.3737 |
| P/Y | 12 |
| C/Y | 12 |

| Solution |
|---|
| $$T_{24} - T_0 = 91\,430.37 - 65\,240.00$$ |
| $$= \$26\,190.37$$ |
| **Specific behaviours** |
| ✓ correct balance after two years |
| ✓ correct increase |

**Question 16** **(8 marks)**

The records of 267 people who were hospitalised with an injury following a road accident have been categorised by road user group and main body region injured in the table below.

|  | Neck | Head | Thorax | Shoulder |
|---|---|---|---|---|
| Motorcyclist | 2 | 8 | 7 | 30 |
| Car occupant | 53 | 66 | 61 | 40 |

(a) Determine what percentage of those hospitalised were car occupants. (1 mark)

| Solution |
|---|
| $53 + 61 + 66 + 40 = 220$ |
| $220 \div 267 = 82\%$ |
| **Specific behaviours** |
| ✓ correct percentage |

(b) State the most common main body region injured by car occupants and what percentage of car occupants had this body region recorded as their main injury. (2 marks)

| Solution |
|---|
| Head, for $66 \div 220 = 30\%$ of car occupants. |
| **Specific behaviours** |
| ✓ correct region |
| ✓ correct percentage |

(c) Complete the following table of row percentages, rounding to the nearest whole number. (3 marks)

| (%) | Neck | Head | Thorax | Shoulder |
|---|---|---|---|---|
| Motorcyclist | 4 | 17 | 15 | 64 |
| Car occupant | 24 | 30 | 28 | 18 |

| Solution |
|---|
| See table |
| **Specific behaviours** |
| ✓ at least two correct entries; ✓ one row correct; ✓ all entries correct |

(d) Do the records suggest the presence of an association between the categorical variables? Justify your answer. (2 marks)
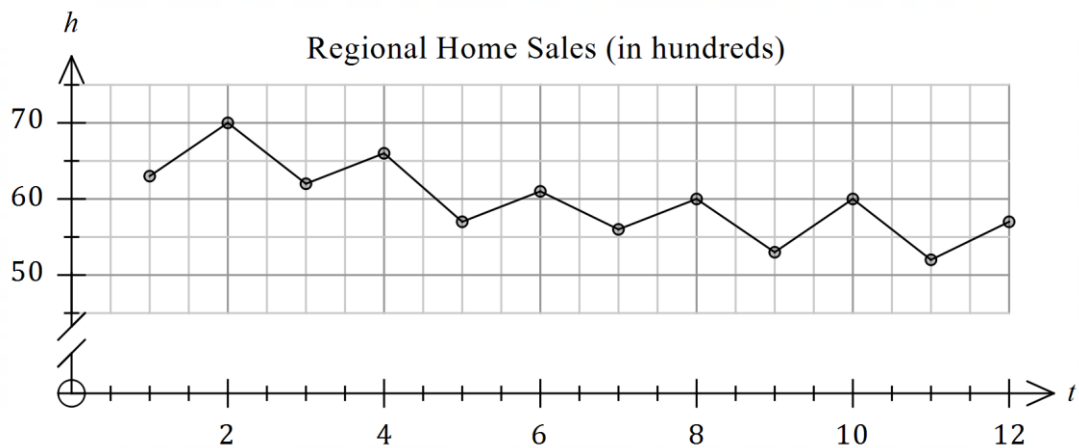
| Solution |
|---|
| Yes, because the percentages in each column (across categories) are quite different. For example, only 4% of motorcyclists had neck as their main injury compared to 24% of car occupants. |
| **Specific behaviours** |
| ✓ states yes to association with justification |
| ✓ observes general differences or uses specific example |

**Question 17** **(10 marks)**

Data for the number of regional home sales per quarter, rounded to the nearest hundred homes, is shown in the table below.

| $t$ | Year | Quarter | Sales ($h$), in hundreds | Quarterly mean | Sales as percentage of quarterly mean |
|---|---|---|---|---|---|
| 1 | | 1 | 63 | | 96.6 |
| 2 | 2015 | 2 | 70 | $A$ | 107.3 |
| 3 | | 3 | 62 | | 95.0 |
| 4 | | 4 | 66 | | 101.1 |
| 5 | | 1 | 57 | | 97.4 |
| 6 | 2016 | 2 | 61 | 58.5 | $B$ |
| 7 | | 3 | 56 | | 95.7 |
| 8 | | 4 | 60 | | 102.6 |
| 9 | | 1 | 53 | | 95.5 |
| 10 | 2017 | 2 | $C$ | | 108.1 |
| 11 | | 3 | 52 | | 93.7 |
| 12 | | 4 | 57 | | 102.7 |



Regional Home Sales (in hundreds)

(a) Describe the trend and seasonality of this data. (2 marks)

| Solution |
|---|
| There is a decreasing trend. |
| Each year, sales tend to be high in Q2 and Q4 and low in Q1 and Q3. |
| **Specific behaviours** |
| ✓ describes trend |
| ✓ describes seasonality |

(b)       Calculate the value of $A$, the value of $B$ and the value of $C$ in the table.        (4 marks)

| Solution |
|---|
| $A = (63 + 70 + 62 + 66) \div 4 = 65.25$ |
| $B = 61 \div 58.5 = 104.3\%$ |
| Let $x$ be quarterly mean for 2017: $53 \div x = 0.955 \Rightarrow x = 55.5$ |
| $C \div 55.5 = 108.1 \Rightarrow C = 60$<br>or<br>$(53 + C + 52 + 57) \div 4 = 55.5 \Rightarrow C = 60$ |
| **Specific behaviours** |
| ✓ value of $A$<br>✓ value of $B$<br>✓ value of quarterly mean<br>✓ value of $C$ |

(c)       Determine the deseasonalised number of home sales in the region in the third quarter of 2015.        (3 marks)

| Solution |
|---|
| $SI_{Q3} = (95.0 + 95.7 + 93.7) \div 3 = 0.9481$ |
| $62 \div 0.9481 = 65.4$ |
| The deseasonalised number of sales is $6\,540$ homes. |
| **Specific behaviours** |
| ✓ calculates seasonal index<br>✓ divides sales by index<br>✓ value that rounds to $6\,500$ and allows for hundreds |

(d)       Forecast the actual number of home sales in the region for the third quarter of 2018 given that the least-squares trend line for the deseasonalised data indicates that the number of home sales to be $5\,070$ at that time.        (1 mark)

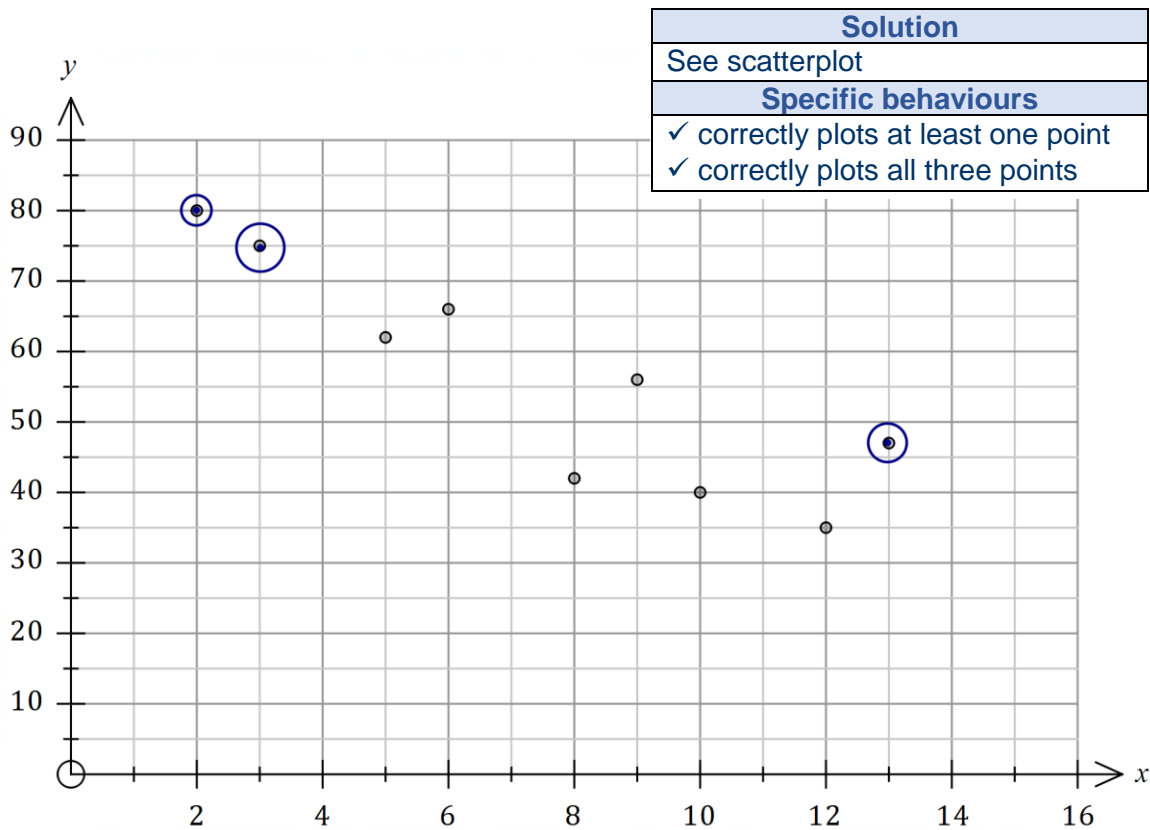| Solution |
|---|
| $h = 5\,070 \times 0.9481 = 4\,800$ |
| **Specific behaviours** |
| ✓ value that rounds to $4\,800$ |

**Question 18**            **(11 marks)**

The table below shows the percentage of all trips made using a bicycle $x$ and a car $y$ for nine countries. The correlation coefficient for the set of data is $-0.89$.

| Country | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| Bicycle trips, $x$ (%) | 5 | 2 | 13 | 3 | 10 | 8 | 12 | 6 | 9 |
| Car trips, $y$ (%) | 62 | 80 | 47 | 75 | 40 | 42 | 35 | 66 | 56 |

(a)     Add the three missing points to the scatterplot below.          (2 marks)



| Solution |
|---|
| See scatterplot |
| **Specific behaviours** |
| ✓ correctly plots at least one point |
| ✓ correctly plots all three points |

(b)     A journalist discussed the dataset in an article with the headline "Decreasing bicycle use causes increase in car use". Comment on their choice of headline.      (2 marks)

| Solution |
|---|
| The headline is misleading as the observed negative association between the variables does not necessarily mean that a change in car use is caused by a change in bicycle use. |
| **Specific behaviours** |
| ✓ indicates that headline is misleading/inappropriate/etc |
| ✓ comment(s) related to causation |

(c)     Determine the equation of the least-squares line with $x$ as the explanatory variable.

(2 marks)

| Solution |
| --- |
| $y = -3.71x + 83.9$ |
| **Specific behaviours** |
| ✓ slope to at least 2 dp |
| ✓ intercept to at least 1 dp |

(d)     In the context of the question, interpret

  (i)     the intercept of the least-squares line.                                                                   (1 mark)

| Solution |
| --- |
| In a country where no trips were made by bicycle, close to $84\%$ of trips would be made using a car. |
| **Specific behaviours** |
| ✓ interpretation using intercept |

  (ii)     the slope of the least-squares line.                                                                   (1 mark)

| Solution |
| --- |
| For every $1\%$ increase in the percentage of trips made by bicycle, there is an observed decrease of $3.7\%$ in the percentage of trips made by car. |
| **Specific behaviours** |
| ✓ reasonable interpretation of negative gradient |

(e)     In country $K$, $16\%$ of all trips are made by bicycle. Predict the percentage of trips made using a car in this country and discuss factors related to the use of the fitted line that affect your confidence in this prediction.                                                                   (3 marks)

| Solution |
| --- |
| $y = -3.71(16) + 83.9 \approx 25\%$ |
| The strong correlation coefficient of $-0.89$ would usually lead to high confidence but due to the large amount of extrapolation, confidence in the prediction is low. |
| **Specific behaviours** |
| ✓ correct prediction (that rounds to given value) |
| ✓ discusses strong correlation |
| ✓ discusses extrapolation |

**Question 19** **(7 marks)**

The balance of a savings account $A_n$ after $n$ monthly deposits have been made can be modelled by $A_{n+1} = 1.012A_n + 180$, $A_0 = 1\,500$.

(a) Determine

(i) the amount deposited each month. (1 mark)

| Solution |
|---|
| $180 |
| **Specific behaviours** |
| ✓ correct amount |

(ii) the annual interest rate of the savings account. (1 mark)

| Solution |
|---|
| $0.012 \times 100 \times 12 = 14.4\%$ p.a. |
| |
| **Specific behaviours** |
| ✓ correct rate |

(iii) the balance of the savings account after 6 monthly deposits have been made.

| N | 6 |
|---|---|
| I% | 14.4 |
| PV | 1500 |
| PMT | 180 |
| FV | -2724.215397 |
| P/Y | 12 |
| C/Y | 12 |

| Solution |
|---|
| $A_6 = \$2\,724.22$ |
| |
| **Specific behaviours** |
| ✓ correct amount |

(1 mark)

After the $300^{\text{th}}$ monthly deposit, no further deposits are made.

(b) Calculate the total interest that the savings account received up to this time. (2 marks)

Amortization

| PM1 | 1 |
|---|---|
| PM2 | 300 |
| I% | 14.4 |
| PV | 1500 |
| PMT | 180 |
| P/Y | 12 |
| C/Y | 12 |
| BAL | |
| INT | |
| PRN | |
| ΣINT | 520568.3372 |
| ΣPRN | |

| Solution |
|---|
| $I = A_{300} - 1\,500 - 300 \times 180$ |
| $\quad = 576\,068.34 - 1\,500 - 54\,000$ |
| $\quad = \$520\,568.34$ |
| |
| **Specific behaviours** |
| ✓ calculates $A_{300}$ |
| ✓ correct method and amount |

| N | 300 |
|---|---|
| I% | 14.4 |
| PV | 1500 |
| PMT | 180 |
| FV | -576068.3372 |
| P/Y | 12 |
| C/Y | 12 |

The accumulated balance still attracts interest, compounded monthly at the same rate, and is used to fund an annual perpetuity.

(c) Determine the amount of the annual perpetuity. (2 marks)

| Solution |
|---|
| $Q = (1.012^{12} - 1) \times 576\,068.34$ |
| $\quad = \$88\,653.82$ |
| |
| **Specific behaviours** |
| ✓ calculates effective interest rate |
| ✓ correct method and amount |

**See next page**

**Question 20** **(7 marks)**

Some consecutive terms of sequence $A$ are shown in the following table.

| $n$ | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|
| $A_n$ | 55 | 49 | 43 | 37 | 31 |

(a) State the name given to this type of sequence and determine $A_1$, the first term of the sequence. **(2 marks)**

| Solution |
|---|
| Arithmetic sequence. |
| $55 = a + 3(-6) \Rightarrow a = A_1 = 73$ |

| Specific behaviours |
|---|
| ✓ name of sequence |
| ✓ first term |

(b) Determine a rule for the $n^{\text{th}}$ term of sequence $A$ in the form $A_n = m \times n + c$, where $m$ and $c$ are both constants. **(2 marks)**

| Solution |
|---|
| $A_n = 73 + (n-1)(-6)$ |
| $= 73 - 6n + 6$ |
| $= -6n + 79$ |

| Specific behaviours |
|---|
| ✓ uses rule on formula sheet |
| ✓ simplifies and states in required form |

The $n^{\text{th}}$ term of sequence $B$ is $B_n$, so that $B_4 = A_{17}$ and $B_{10} = A_{12}$.

(c) Determine $B_{14}$. **(3 marks)**

| Solution |
|---|
| $B_4 = A_{17} = -23, \qquad B_{10} = A_{12} = 7$ |
| $6d = 7 - (-23) = 30 \Rightarrow d = 5$ |
| $B_{14} = B_{10} + 4d = 7 + 4(5) = 27$ |

| Specific behaviours |
|---|
| ✓ recognises $A_{12}$ and $A_{17}$ values represent $B_{10}$ and $B_4$ |
| ✓ finds a common difference $d$ |
| ✓ correct $B_{14}$ term |

**Question 21**                                                **(7 marks)**

A company took out a business loan of $165 000 at an interest rate of 9.66% per annum and made monthly repayments of $5 300. The first few rows of a spreadsheet used by the company to track the loan balance is shown below.

| Month, $n$ | Balance at start of month $n$ | Interest | Repayment | Balance carried forward |
|---|---|---|---|---|
| 1 | 165 000.00 | 1 328.25 | 5 300.00 | 161 028.25 |
| 2 | 161 028.25 | 1 296.28 | 5 300.00 | 157 024.53 |
| 3 | 157 024.53 | $A$ | 5 300.00 | $B$ |

(a)    Determine the value of $A$ and the value of $B$ in the spreadsheet.          (2 marks)

| Solution |
|---|
| $A = 157\,024.53 \times 0.0966 \div 12 = 1\,264.05$ |
| $B = 157\,024.53 + 1\,264.05 - 5\,300 = 152\,988.57$ |
| **Specific behaviours** |
| ✓ value of $A$ |
| ✓ value of $B$ |

A recurrence relation of the form $T_{n+1} = rT_n - d$, $T_1 = a$ can be used to model the balance of the loan at the start of month $n$.

(b)    Determine the value of each of the constants $r, d$ and $a$ in the recurrence relation.

                                                                           (2 marks)

| Solution |
|---|
| $r = 1 + (9.66 \div 12 \div 100) = 1.00805$ |
| $d = 5\,300,$      $a = 165\,000$ |
| **Specific behaviours** |
| ✓ value of $r$ |
| ✓ value of $d$ and value of $a$ |

(c)    Using a financial calculator, or otherwise, state
(i)     the balance of the loan after 12 repayments. (1 mark)

| Solution |
|---|
| $115 171.15 |
| **Specific behaviours** |
| ✓ correct value |

| N | 12 |
|---|---|
| I% | 9.66 |
| PV | 165000 |
| PMT | −5300 |
| FV | −115171.1468 |
| P/Y | 12 |
| C/Y | 12 |

(ii)    the number of repayments required to repay the loan.
(1 mark)

| Solution |
|---|
| 36 months. |
| **Specific behaviours** |
| ✓ correct value |

| N | 35.98256867 |
|---|---|
| I% | 9.66 |
| PV | 165000 |
| PMT | −5300 |
| FV | 0 |
| P/Y | 12 |
| C/Y | 12 |

(iii)    the minimum monthly repayment for the full amount of the loan to be repaid at the same interest rate in 24 equal repayments.
(1 mark)

| Solution |
|---|
| $7 588.05 |
| **Specific behaviours** |
| ✓ correct value |

| N | 24 |
|---|---|
| I% | 9.66 |
| PV | 165000 |
| PMT | −7588.045975 |
| FV | 0 |
| P/Y | 12 |
| C/Y | 12 |

**End of questions**

Supplementary page

Question number: _____